ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2018, Vol. 39, No. 2, pp. 286–288. © Pleiades Publishing, Ltd., 2018.

Special Metric Invariants

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(Submitted by M. A. Malakhaltsev)

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Received February 2, 2017

Abstract—In the present paper we obtain new metric invariants of metric spaces. These invariants can be used for classification of the finite metric spaces, their recognition and in the research of Tammes' problem for sphere.

DOI: 10.1134/S1995080218020269

Keywords and phrases: *finite metric space, metric invariant, main metric invariant.*

1. INTRODUCTION

Let $\mathbb{N}$ ($\mathbb{R}_+$) be a set of all natural (nonnegative real) numbers. Let us consider a set $\mathbb{K}$ of metric spaces to have the same power $N > 1$ and let us remind you about the definition of the main metric invariant [1].

Definition 1. *A function $F : \mathbb{K} \to \mathbb{R}_+$ is called a main metric invariant, if the following conditions are satisfied:*

(i) $F(X) = F(Y)$ for any isometric metric spaces X, $Y \in \mathbb{K}$; in this case the function F is called a metric invariant;

(ii) $F(X) \in \{\rho(x, y) : x, y \in X, x \neq y\}$ for any metric space $(X, \rho) \in \mathbb{K}$;

(iii) $|F(X) - F(Y)| \leq 2d_s(X, Y) = \inf\{disf : f : X \to Y is\ bijection\}$, where $disf = \sup\{|\rho(x, y) - d(f(x), f(y))| : x, y \in X\}$, for all metric spaces (X, ρ), $(Y, d) \in \mathbb{K}$.

We will give the known two examples of main metric invariants of a metric space.

1. Diameter of a space $X \in \mathbb{K}$: $\mathrm{D}(X) = \sup\{\rho(x, y) : x, y \in X\}$ [2, 3, p. 305].

2. K-radii, where $K \in \mathbb{N}$, of a space X:

$$\mathrm{R}_K(X) = \inf\{\beta(X, S) : S \subset X, 1 \leq \text{card } (S) \leq K\}, \tag{1}$$

where $\beta(X, S) = \sup\{\rho(x, S) : x \in X\}$, card (S) is a power of the set S ([2], where notation $\mathrm{R}_{KX}(X) = \mathrm{R}_K(X)$ was used). The other examples can be found in [1]. The main metric invariants can be used for classification of the finite metric spaces and their recognition. Let us introduce the following

Definition 2. *Let $S \subset X$,* card $(S) = k$, where $k \in \mathbb{N}\backslash\{1\}$,

$$m(S) = \min\{\rho(x, y) : x, y \in S, x \neq y\}, \quad \sigma(S) = \frac{1}{k(k-1)} \sum_{x,y \in S} \rho(x, y).$$

We introduce the following metric invariants:

$$\varepsilon_k(X) = \sup\{m(S) : S \subset X, \text{card } (S) = k\},$$
$$\mathrm{P}_k(X) = \sup\{\sigma(S) : S \subset X, \text{card } (S) = k\}.$$

Note that $\varepsilon_2(X) = \mathrm{D}(X)$ for every $X \in \mathbb{K}$. The invariant $\varepsilon_k(X)$ in the case of finite metric space in [4, 1] was denoted $\mathrm{mar}_{(k-1)k}(X)$. Other its properties and application in the geometry of trees of finite

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